Static Pushover Analysis Based on an Energy-Equivalent SDOF System

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In this paper, a new energy-based pushover procedure is presented in order to achieve an approximate estimation of structural performance under strong earthquakes. The steps of the proposed methodology are quite similar to those of the well-known displacement modification method. However, the determination of the characteristics of the equivalent single-degree-of-freedom (E-SDOF) system is based on a different rational concept. Its main idea is to determine the E-SDOF system by equating the external work of the lateral loads acting on the multi-degree-of-freedom (MDOF) system under consideration to the strain energy of the E-SDOF system. After a brief outline of the theoretical background, a representative numerical example is given. Finally, the accuracy of the proposed method is evaluated by an extensive parametric study which shows that, in general, it provides better results compared to those produced by other similar procedures. [DOI: 10.1193/1.3535597]

INTRODUCTION

In the last decades, many research efforts have focused on developing simple procedures for the approximate estimation of the inelastic performance of buildings under seismic excitation, in order to avoid the significant computational cost and the various inherent disadvantages of an accurate inelastic dynamic analysis. As a result of these efforts, the idea of pushover analysis has been born. Recently, a series of more or less similar inelastic static pushover procedures have been developed, some of which have been already adopted by several seismic codes and prestandards (ASCE 41-06, ATC-40, EC-8, etc.). All of these procedures are based on the assumption that the inelastic response of a structure can be related to the response of an equivalent single-degree-of-freedom (E-SDOF) system. As a first step, the structure is subjected to incremental lateral forces with constant distribution along the height and the base shear versus roof displacement diagram is plotted (capacity or pushover curve). The capacity curve is then idealized to a bilinear curve from which the fundamental properties of an E-SDOF system are determined. On the basis of several additional simplifying assumptions, the peak roof displacement of the structure (target displacement) is correlated to the peak response of the E-SDOF system which is determined with the aid of a selected design or response spectrum. All other response quantities are determined by conducting pushover analysis up to the already calculated target displacement.

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Static pushover analysis, or nonlinear static procedure (NSP) as it is referred in seismic codes, seems to be a useful tool for engineering practice. Nevertheless, it has already been stressed by many researchers (e.g., Krawinkler and Seneviratna 1998) that this procedure involves many shortcomings and can provide reasonable results only for low- and medium-rise planar systems. This is mainly due to the fact that the determination of the structure's response is based on the assumption that the dynamic behavior depends only on a single elastic vibration mode. In addition, this elastic mode is supposed to remain constant despite the successive formation of plastic hinges during the seismic excitation. Also, the choice of roof displacement instead of any other displacement is arbitrary and it is doubtful whether the capacity curve is the most meaningful index of the nonlinear response of a structure, especially for irregular and spatial systems. Thus, many researchers have proposed modified pushover procedures to overcome these shortcomings (e.g., Chopra and Goel 2001, Hernadez-Montes et al. 2004, Parducci et al. 2006, Oliveto et al. 2001). Some of them (Hernadez-Montes et al. 2004, Parducci et al. 2006, Oliveto et al. 2001) are based on the energy equivalence between the multi-degree-of-freedom (MDOF) and the E-SDOF systems (energy-based procedures). According to the energy-based procedures, the strain energy of the structure or, equivalently, the work done by the external loads is considered to be the most representative index of its nonlinear response.

In order to account for the higher modes contribution to the nonlinear dynamic response of structures, Chopra and Goel (2001) introduced modal pushover analysis (MPA). MPA comprises a series of static pushover analyses, one for each of the vibration modes taken into account. However, the capacity curves of higher modes often present disproportionate increases and even outright reversals of roof displacements. To avoid this trouble, Hernadez-Montes et al. (2004) suggested an energy-based formulation of pushover analysis which uses a target-displacement derived from the work done by the lateral loads to establish the capacity curve, instead of using the roof displacement. In each step of the pushover procedure, the work done by lateral loads associated with each mode is computed using an incremental formulation. The corresponding increment in the energy-based displacement is calculated by dividing the increment of work at each step by the base shear at that step. The incremental displacements are accumulated to obtain the energy-based displacement of the E-SDOF system. Thus, a modified capacity curve is plotted for each mode, which is used in place of the conventional pushover curve. These modified curves resemble traditional first mode pushover curves and correct the anomalies observed in higher mode curves.

Parducci et al. (2006) proposed the determination of an equivalent energy-based displacement of the E-SDOF system. This displacement does not correspond to any actual point of the structural model, but it is a virtual value equalizing the work done by the lateral loads to the strain energy of the E-SDOF system. Then, a strain energy versus equivalent displacement diagram is plotted and in combination with a pseudo-energy response spectrum, a performance point of the structure is estimated.

Earlier, Oliveto et al. (2001) determined a displacement parameter based on power equivalence (which in finite terms translates into energy equivalence) between MDOF and E-SDOF systems. The properties of the E-SDOF system are then calculated as function of this energy-based displacement. Recently, this procedure was extended to include Modal Pushover Analysis (Biondi and Oliveto 2008).

The objective of this paper is the presentation and preliminary evaluation of a new energy-based NSP for the approximate estimation of the seismic response of structures. This procedure uses the strain energy which is considered as a more meaningful index of the structural response than the base shear. This is due to the fact that the strain energy depends on the values of all forces acting to the structure as well as on the values of the displacements of all the system's degrees of freedom. The steps of the proposed methodology are quite similar to those of the well-known displacement modification method (ASCE 41-06, EC-8). However, the determination of the characteristics of the E-SDOF system is based on a different concept. Specifically, the definition of the E-SDOF system is based on the equalization of the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system. In contrast to other energy-based procedures, the energy equivalence is used to derive a modified resisting force of the E-SDOF system, instead of an energy-based displacement. Thus, a modified capacity curve is plotted. This curve is consistent with the strain energy versus displacement diagram of the E-SDOF system and it is used for the establishment of the E-SDOF system. As a first step, the procedure is formulated in a manner that takes into account only the predominant vibration mode and in its current form it can be rigorously applied to low- and medium-rise planar systems. Firstly, the theoretical background and the assumptions of the proposed methodology are presented and briefly discussed. Taking into account the basic assumptions and applying well-known principles of structural dynamics, some fundamental conclusions are derived and, on that basis, an alternative, energy-equivalent SDOF system is established, which can be used for the estimation of the target displacement. Secondly, both steps needed for the implementation of the proposed methodology along with the necessary equations are systematically presented. In order to facilitate comprehension, a clarifying numerical example is given. Finally, the accuracy of the proposed methodology is evaluated by an extensive parametric study. The whole investigation shows that, in general, it gives better results compared to those produced by other similar procedures. The paper closes with comments on results and conclusions.

INELASTIC RESPONSE OF MDOF SYSTEM

DECOMPOSITION TO RESPONSES OF SDOF SYSTEMS

The response of a MDOF system with N degrees of freedom to an earthquake ground motion $\ddot{u}_g(t)$ is governed by the following equation:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{F}_s(t) = -\boldsymbol{M}\boldsymbol{\delta}\ddot{u}_g(t) \tag{1}$$

where $\boldsymbol{u}(t)$ is the displacement vector of the N degrees of freedom (translations or rotations) relative to the ground, $\boldsymbol{M}$ is the NxN diagonal mass matrix, $\boldsymbol{C}$ is the NxN symmetric damping matrix, $\boldsymbol{F}_s$ is the vector of the resisting forces (or moments), i.e., the forces that would have to be applied to the structure in order to obtain displacements $\boldsymbol{u}(t)$ (for the sake of simplicity (t) is left out subsequently) and $\boldsymbol{\delta}$ is the influence vector that describes the influence of support displacements on the structural displacements. The terms of $\boldsymbol{\delta}$ corresponding to translational degrees of freedom parallel to the excitation direction are equal to unity, while the rest are equal to zero. In the linear elastic range of behavior the response can be decomposed to responses of SDOF systems, one for each elastic vibration mode (modal analysis).

In the inelastic range of behavior some basic assumptions have to be made, keeping always in mind that our main intention and aim is the development of an approximate, simplified nonlinear static procedure. A major assumption is that the response of a MDOF system can be expressed as superposition of the responses of appropriate SDOF systems just like in the linear range. Of course, such an assumption violates the very logic of nonlinearity, as the superposition principle does not apply in nonlinear systems. However, it must be thought as a fundamental postulate, which constitutes the basis on which many simplified pushover procedures are built. Thus, each SDOF system corresponds to a vibration "mode" i with "modal" vector $\boldsymbol{\varphi}_i$ (the quotation marks indicate that the application of the superposition principle is not strictly valid). The displacements $\boldsymbol{u}_i$ and the inelastic resisting forces $\boldsymbol{F}_{si}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\boldsymbol{M\varphi}_i$ respectively. Furthermore, "modal" vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite the successive development of plastic hinges. With the aforementioned assumptions, the vectors $\boldsymbol{u}$ and $\boldsymbol{F_s}$ can be expressed as sum of "modal" contributions as follows (Anastassiadis 2004, Chopra 2007):

$$\boldsymbol{u} = \sum_{i=1}^{N} \boldsymbol{u}_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i q_i \tag{2}$$

$$\boldsymbol{F}_s = \sum_{i=1}^{N} \boldsymbol{F}_{si} = \sum_{i=1}^{N} \alpha_i \boldsymbol{M} \boldsymbol{\varphi}_i \tag{3}$$

where $\boldsymbol{\alpha}_i$ is a hysteretic function that depends on the "modal" co-ordinate q_i and the history of excitation (Anastassiadis 2004). The quantity:

$$V_i = \boldsymbol{\delta}^T \boldsymbol{F}_{si} = \boldsymbol{\delta}^T \alpha_i \boldsymbol{M} \boldsymbol{\varphi_i} = \alpha_i L_i \tag{4}$$

where $L_i = \boldsymbol{\delta}^T \boldsymbol{M\varphi}_i$, represents the sum of "modal" loads corresponding to non zero terms of vector $\boldsymbol{\delta}$, i.e., in the usual case of horizontal excitation, V_i is equal to the "modal" base shear parallel to the direction of excitation. By substituting Equations 2 and 3 into Equation 1, premultiplying both sides of Equation 1 by $\boldsymbol{\varphi}_i^T$ and using the orthogonality property of "modes," N uncoupled equations can be derived:

$$M_i\ddot{q}_i + 2M_i\omega_i\zeta_i\dot{q}_i + M_i\alpha_i = -L_i\ddot{u}_g \Leftrightarrow \ddot{q}_i + 2\omega_i\zeta_i\dot{q}_i = \alpha_i = -v_i\ddot{u}_g \tag{5}$$

where $M_i = \boldsymbol{\varphi}_i{}^T \boldsymbol{M\varphi}_i$, ζ_i and $v_i = L_i/M_i$ are the generalized mass, the damping ratio or fraction of critical damping (it is supposed that Rayleigh damping is present) and the modal participation factor of vibration mode i respectively. Substituting $q_i = v_iD_i$ into Equation 5 and multiplying both sides by L_i gives:

$$L_iv_i\ddot{D}_i + L_i2\omega_i\zeta_iv_i\dot{D}_i + L_i\alpha_i = -L_iv_i\ddot{u}_g \Leftrightarrow M_i^*\ddot{D}_i + 2M_i^*\omega_i\zeta_i\dot{D}_i + V_i = -M_i^*\ddot{u}_g \tag{6}$$

where $M_i^* = v_i\, L_i$ is the effective modal mass of mode i. Equation 6 shows that, due to the aforementioned assumptions, the nonlinear response of a MDOF system with N degrees of freedom subjected to an horizontal earthquake ground motion $\ddot{u}_g$ can be expressed as the sum of the responses of N SDOF systems, each one corresponding to a vibration "mode" having mass equal to M_i^*, displacement equal to D_i and inelastic resisting force equal to V_i. Obviously, this definition of the SDOF systems is not unique, e.g., the mass could be taken equal to unity and the resisting force equal to the quantity V_i/M_i^*. However, according to the authors the definition presented above is the most convenient one.

EXTERNAL WORK OF "MODAL" FORCES $\boldsymbol{F_{si}}$

A MDOF system with N degrees of freedom which is subjected in the differential time interval dt to an excitation $\ddot{u}_g$ has the differential displacements:

$$d\boldsymbol{u} = \sum_{i=1}^{N} d\boldsymbol{u}_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i dq_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i v_i dD_i \tag{7}$$

The incremental work of "modal" forces $\boldsymbol{F}_{si}$ of "mode" i on the displacements $d\boldsymbol{u}_i$ can be written as:

$$dE_i = \sum_{j=1}^{N} du_{ji} F_{ji} \tag{8}$$

where du_{ji} and F_{ji} are the j-elements of vectors $d\boldsymbol{u}_i$ and $\boldsymbol{F}_{si}$ respectively (Figure 1). Writing Equation 8 in matrix form and using Equations 2, 3, and 4 gives:

$$\begin{aligned} &dE_i = d\boldsymbol{u}_i^T \boldsymbol{F}_{si} \Rightarrow dE_i = \boldsymbol{\varphi}_i^T v_i dD_i \alpha_i \boldsymbol{M} \boldsymbol{\varphi}_i \Rightarrow dE_i = \alpha_i v_i dD_i (\varphi_i^T \boldsymbol{M} \boldsymbol{\varphi}_i) \Rightarrow \\ &dE_i = \alpha_i \frac{L_i}{M_i} dD_i M_i \Rightarrow dE_i = \alpha_i L_i dD_i \Rightarrow dE_i = V_i dD_i \end{aligned} \tag{9}$$

Equation 9 shows that the external work of "modal" forces $\boldsymbol{F}_{si}$ on the displacements $d\boldsymbol{u}_i = v_i \boldsymbol{\varphi}_i dD_i$ is equal to the work of the resisting force (or the strain energy) of the corresponding SDOF system for the displacement dD_i.

CHARACTERISTICS OF INELASTIC SDOF SYSTEMS

An inelastic SDOF system is usually described by a bilinear force-displacement diagram V-D (Figure 2a), from which its most important characteristics can be derived. For the implementation of NSPs the characteristics of interest are the natural period T and the yield strength reduction factor R. The calculation of T and R is carried out successively as follows:

$$T = 2\pi\sqrt{\frac{mD_y}{V_y}} \rightarrow S_a(T) \rightarrow \mathrm{R} = \frac{mS_a(T)}{V_y} \tag{10}$$

where m, D_y, V_y are the mass, the yield displacement and the yield strength of the system respectively and $S_a(T)$ is the spectral acceleration. Alternatively, the behavior of an inelastic SDOF can be described by a strain energy-displacement diagram E-D (Figure 2b) and the characteristics of interest can be derived from Equations 11 and 12 (where $S_d(T)$ is the

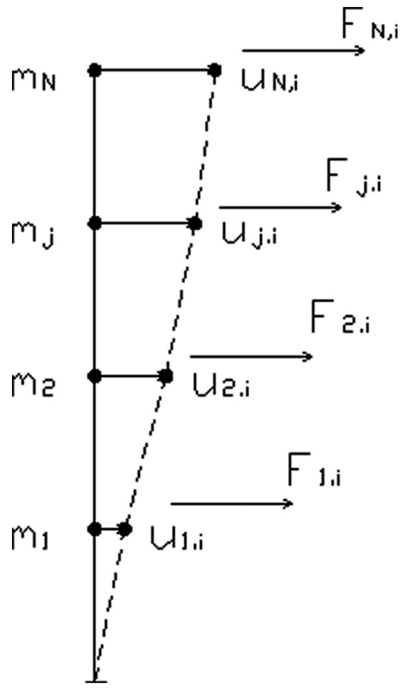


Figure 1. Modal displacements u_{ji} and modal forces F_{ji} for mode i.

spectral displacement). The *E-D* diagram is a second-degree parabolic curve in the linear range ($E = \frac{1}{2}k\ D^2$), while in the nonlinear range is a superposition of a parabola and a straight line [$E = E_{el} + \frac{1}{2}\alpha k\ (D\text{-}D_y)^2 + V_y\ (D\text{-}D_y)$]. In the special case of an elastic-perfectly plastic system ($\alpha = 0$) the curve degenerates to a straight line with slope V_y (dashed line in Figure 2b). The two alternative ways of describing the behavior of SDOF system are absolutely equivalent.

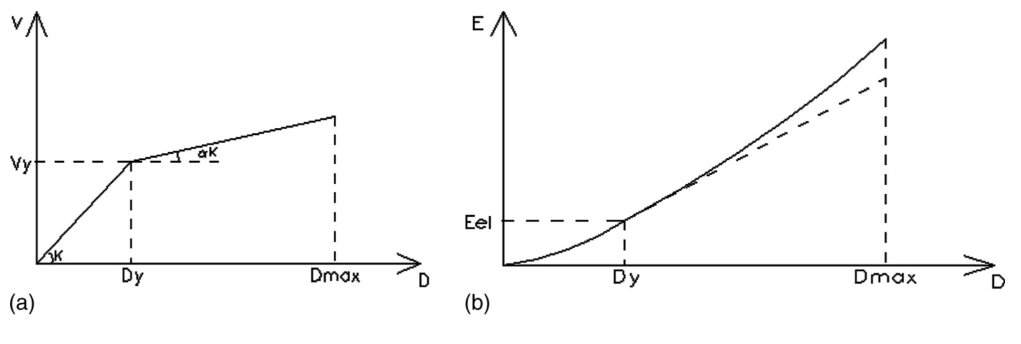


Figure 2. (a) Force-displacement *V-D* curve, and (b) strain energy-displacement *E-D* curve.

$$E_{el} = \frac{1}{2} V_y D_y = \frac{1}{2} k D_y^2 \tag{11}$$

$$T = 2\pi \sqrt{\frac{m D_y^2}{2E_{el}}} \rightarrow S_a(T) \rightarrow S_d(T) \rightarrow R = \frac{S_d(T)}{D_y} \tag{12}$$

THE PROPOSED METHODOLOGY

From the analysis presented above, some basic equations that correlate the properties of the “modal” E-SDOF systems to the properties of the MDOF system are derived and summarized in Table 1. However, these equations are derived on the basis of the aforementioned assumptions and cannot be true all together when a pushover analysis is conducted. Thus, Modal Pushover Analysis (Chopra and Goel 2001) leaves out the third equation and uses the two others to establish the “modal” E-SDOF systems. The conventional procedures adopted by codes follow a similar approach with some additional assumptions. In particular, they take into account only the predominant vibration mode and they permit modifications to the corresponding mode shape vector. The existing energy-based single or multimodal procedures keep the last two equations and determine the E-SDOF systems’ displacements from the energy equivalence between them and the MDOF system. Nevertheless, it must be stated that these two equations are derived as a consequence of the validity of the first. In fact, the modification of roof displacement violates the main assumptions the entire procedure is based. On the contrary, the proposed method keeps the first and the third equations and uses the energy equivalence to determine a modified resisting force of the E-SDOF system. This concept is more consistent with the aforementioned fundamental assumptions. As a first step, the proposed method is formulated in a manner that takes into account only the predominant vibration mode in the excitation direction, so in its current form it is suitable for structural systems with small contribution of higher modes, such as low- and medium-rise planar frames.

The steps needed for the implementation of the proposed methodology are as follows:

Step 1. Create the structural model.

Step 2. Apply to the model a set of lateral incremental forces proportional to the vector $\boldsymbol{M\varphi}_1$ of the fundamental elastic vibration mode *1* (Figure 1) and determine the strain energy-displacement curve E_1-u_{N1}. u_{N1} can be chosen to correspond to any degree of freedom, but

Table 1. Definition of the E-SDOF systems

MDOF System		E-SDOF Systems
“modal” displacements $\boldsymbol{u}_i^{\mathrm{T}} = \boldsymbol{\varphi}_i^{\mathrm{T}} \mathrm{v}_i\, D_i$ (roof displacement u_{Ni})	$\Rightarrow$	displacement $D_i = u_{Ni}/\mathrm{v}_i\, \boldsymbol{\varphi}_{Ni}$ (1st)
“modal” base shear V_i	$\Rightarrow$	resisting force $V_{SDOFi} = V_i$ (2nd)
work of “modal” forces on the differential “modal” displacements $d\boldsymbol{u}_i^{\mathrm{T}} = \boldsymbol{\varphi}_i^{\mathrm{T}} \mathrm{v}_i\, dD_i\, E(d\boldsymbol{u}_i)$	$\Rightarrow$	work of resisting force on the differential displacement dD_i $E(dD_i) = E(d\boldsymbol{u}_i)$ (3rd)

usually the roof displacement parallel to the excitation direction is used. E_1 is equal to the work of the external forces. In the linear range the E_1-u_{N1} diagram is a parabolic curve and if the $\boldsymbol{\varphi}_1$ vector is normalized to u_{N1} (i.e., $\varphi_{N1} = 1$), the strain energy is given by Equation 13:

$$E_{el,1} = \frac{1}{2}\boldsymbol{u}_1^T\boldsymbol{K}\boldsymbol{u}_1 = \frac{1}{2}u_{\mathrm{N}_i}\boldsymbol{\varphi}_1^T\boldsymbol{K}\boldsymbol{\varphi}_1 u_{N1} = \frac{1}{2}k_1 u_{N1}^2 \tag{13}$$

where k_1 is the generalized stiffness of mode *1*, i.e., the stiffness that would have the linear elastic SDOF system corresponding to elastic vibration mode *1*. In the inelastic range the E_1-u_{N1} diagram is gradually created by superposition of lines and parabolic curves with discontinuities of curvature at the points of creation of plastic hinges.

Step 3. Divide the abscissas of the E_1-u_{N1} diagram by the quantity $v_1\varphi_{N1} = u_{N1}/D_1$ and determine the E_1-D_1 diagram of the E-SDOF system (Figure 3b). By utilizing a graphic procedure, the E_1-D_1 diagram could be idealized to a smoothed diagram without curvature discontinuities (like the *E-D* diagram of Figure 2b) and the characteristics of the E-SDOF system could be derived directly from Equations 11 and 12. However, because of the complexity of the E_1-D_1 diagram this approach is difficult to apply, so follow step 4.

Step 4. Calculate the work $E_{1,\lambda}$ (Figure 3b) of the external forces in each of λ discrete intervals between the successive formation of plastic hinges. $dE_{1,\lambda}$, as part of $E_{1,\lambda}$ (Equation 14), is considered to derive from Equation 15.

$$dE_{1,\lambda} = E_{1,\lambda} - V_{1,\lambda-1}(D_{1,\lambda} - D_{1,\lambda-1}) = E_{1,\lambda} - V_{1,\lambda-1}dD_{1,\lambda} \tag{14}$$

$$dE_{1,\lambda} = \frac{1}{2}k_{1,\lambda}\; dD_{1,\lambda}^2 \Rightarrow k_{1,\lambda} = 2dE_{1,\lambda}/dD_{1,\lambda}^2 \tag{15}$$

where $k_{1,\lambda}$ is the stiffness of the E-SDOF corresponding to mode *1* in the interval λ. The resisting force $V_{1,\lambda}$ is given by Equation 16:

$$V_{1,\lambda} = V_{1,\lambda-1} + k_{1,\lambda}\; dD_{1,\lambda} \tag{16}$$

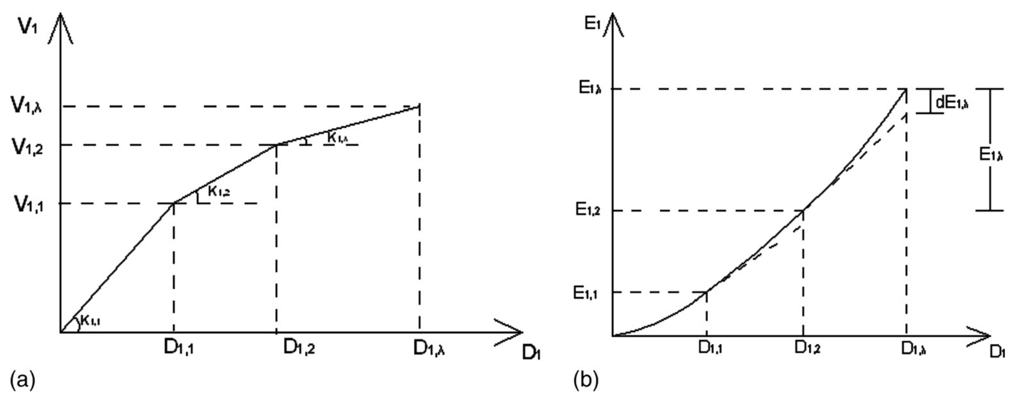


Figure 3. (a) Force-displacement V_1-D_1 curve, and (b) strain energy-displacement E_1-D_1 curve.

For $\lambda = 1$ (i.e., when the first plastic hinge is created) the force $V_{1,1}$ is equal to the base shear parallel to the direction of excitation. By utilizing Equations 14, 15, and 16 for each interval, determine the force-displacement diagram V_1-D_1 of mode *1* (Figure 3a).

Step 5. Idealize V_1-D_1 to a bilinear curve using one of the well known graphic procedures (e.g., ASCE 41-06, Section 3.3.3.2.5) and calculate the period T and the yield strength reduction factor R of the E-SDOF system corresponding to mode *1* from Equation 10. It is stated that the mass m is equal to the effective modal mass M_1^* of mode *1* (Equation 6).

Step 6. Calculate the target displacement and other response quantities of interest (drifts, plastic rotations, etc.) of mode *1*, using one of the well known procedures of displacement modification (e.g., ASCE 41-06, Section 3.3.3.3.2 / *FEMA 440*, Section 10.4). When the procedure is applied for research purposes using recorded earthquake ground motions, it is recommended to estimate the inelastic displacement of the E-SDOF system by means of nonlinear dynamic analysis, instead of using the relevant coefficients (e.g., C_1 in ASCE 41-06 and *FEMA 440*). This is due to the fact that the coefficient values given by codes are based on statistical processing of data with excessive deviation and, therefore, great inaccuracies could result (Manoukas et al. 2006).

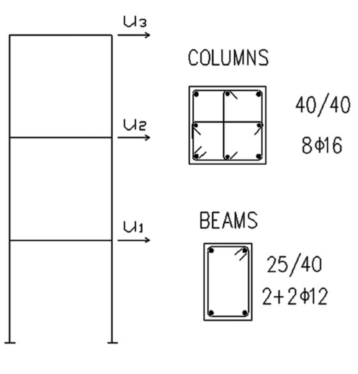
Step 7. Repeat Steps 2 through 6 applying the incremental forces in the opposite direction. It is obvious that this step is necessary to apply only for asymmetric structures.

NUMERICAL EXAMPLE

In order to explain how the proposed methodology (PM) should be applied, an analytical example is illustrated. In particular, PM is applied to a three-story R/C regular planar frame (Figure 4) for the 1940 El Centro NS ground motion multiplied by 0.5, 1.0 and 1.5 and the results are compared with those obtained by nonlinear response history analysis (NL-RHA).

STEP 1. STRUCTURAL MODEL CREATION

Both PM and NL-RHA performed using the program SAP 2000 v10.0.7. The modeling of the inelastic behavior is based on the following assumptions:



Storey Height: *4m* Bay Width: *4m*
Concrete: *C16/20* (f_{ck}*=16 MPa)*
Reinforcement Bars: *S400* (f_{yk}*=400 MPa)*
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic Mass: *m=10t per level (30t total)*
Gravity Loads: not considered

Fundamental Vibration Mode

Damping Ratio: $\zeta_1 = 5\%$
Natural Period: $T_1 = 0.52sec$
$\boldsymbol{\varphi}_1^T = [\ 0.295,\ 0.711,\ 1\ ]$
Modal Participation Factor: $v_1 = 1.26$
Effective Modal Mass: $M_1^* = 25.26t$

Figure 4. Three-story R/C planar frame.

- Shear failure is precluded.
  The inelastic deformations are concentrated at the critical sections, i.e., at the ends of the frame elements (plastic hinges).
- Plastic hinges are modeled by bilinear elastic-perfectly plastic moments-rotations diagrams (M-θ).
- The bending moment-axial force interaction is taken into account by using the ACI 318-02 interaction surface which is available in the program SAP 2000 v10.0.7.

STEP 2. APPLICATION OF FORCES AND DETERMINATION OF E_1-U_{N1} DIAGRAM

The structural model is subjected to horizontal incremental forces with distribution along the height proportional to the vector $\boldsymbol{M\varphi_1}$ of elastic vibration mode 1 ($\boldsymbol{\varphi_1}$ is normalized to the roof displacement). Every time a plastic hinge appears, the floor displacements and forces are recorded and the external work (= strain energy) at each discrete interval between the successive formation of plastic hinges is calculated, so the external work-roof displacement diagram for the first vibration mode E_1-u_{N1} can be plotted as shown in Figure 5.

STEP 3. DETERMINATION OF E_1-D_1 DIAGRAM

By dividing the abscissas of the E_1-u_{N1} diagram by the quantity $u_{N1}/D_1 = v_1\varphi_{N1} = 1.26$ x $1.00 = 1.26$ the strain energy-E-SDOF system displacement diagram for the first vibration mode E_1-D_1 is determined (Figure 5).

STEP 4. DETERMINATION OF V_1-D_1 DIAGRAM

The resisting force $V_{1,\lambda}$ at each step λ is calculated by applying Equations 14, 15, and 16, so the resisting force-E-SDOF system displacement diagram for the first vibration mode V_1-D_1 is determined (Figure 6a). In the same figure the corresponding diagram derived by the conventional pushover procedure (CP) is also plotted.

STEP 5. IDEALIZATION OF V_1-D_1 DIAGRAM AND CALCULATION OF THE CHARACTERISTICS OF THE E-SDOF SYSTEM

The resisting force-E-SDOF system displacement diagram V_1-D_1 is idealized to a bilinear curve (Figure 6b). The idealization is based on the following assumptions:

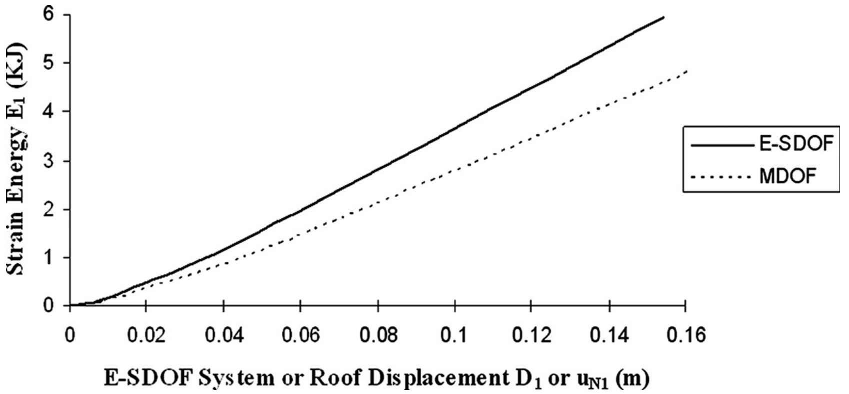
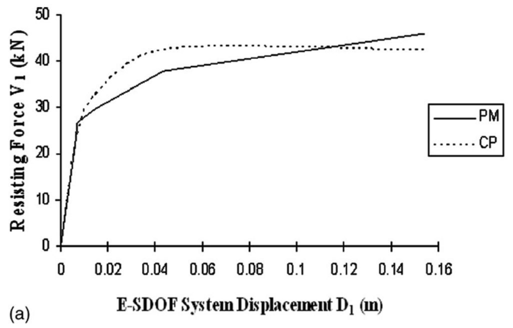


Figure 5. External work-roof displacement E_1-u_{N1} and strain energy-E-SDOF system displacement E_1-D_1 diagrams.



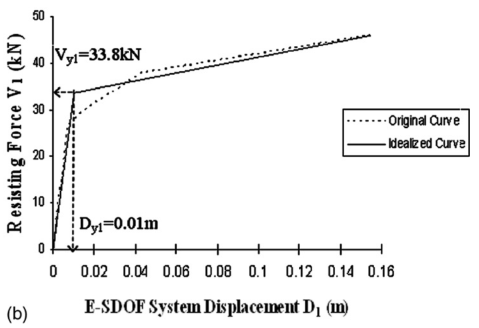


Figure 6. (a) Resisting force-E-SDOF system displacement diagrams V_1-D_1 for the first mode, and (b) idealization of resisting force-E-SDOF system displacement diagram for the proposed method.

- The areas between each curve and displacement axis (i.e., the strain energy of the E-SDOF system) should be equal.
- It is assumed that the original and the idealized curves intersect each other at the maximum displacement.

Of course, one may alternatively utilize another graphic procedure (e.g., ASCE 41-06, Section 3.3.3.2.5). The characteristics of the E-SDOF system are calculated from Equation 10 for each ground motion considered and they are shown in Table 2.

STEP 6. CALCULATION OF TARGET DISPLACEMENT AND OTHER RESPONSE QUANTITIES

The target displacement is calculated by means of NL-RHA of the E-SDOF system and multiplication of the resulting displacement by $v_1\varphi_{N1} = 1.26$. The remaining response quantities are determined by conducting pushover analysis up to the target roof displacement. The results determined by the PM are compared with those obtained by NL-RHA.

Table 3 shows the story displacements and drifts determined by PM and NL-RHA. It becomes clear that the two procedures give similar displacement profiles. However, PM is a little conservative. Specifically, in reference to the roof displacement, which is considered as representative of the seismic performance of structures, PM leads to an error from about 1% (1.0 x El Centro NS) to 40% (1.5 x El Centro NS). The story drifts are also

Table 2. Characteristics of the E-SDOF system

Ground Motion	Period *T (sec)*	Yield Strength Reduction Factor *R*
0.5 x El Centro NS	0.543	3.18
1.0 x El Centro NS	0.543	6.36
1.5 x El Centro NS	0.543	9.54

Table 3. Floor displacements, story drifts and plastic rotations at critical sections of beams

	Floor Displacements (cm)					
	0.5 x El Centro NS		1.0 x El Centro NS		1.5 x El Centro NS	
Story	PM	NL-RHA	PM	NL-RHA	PM	NL-RHA
1	0.800	0.706	1.199	1.004	2.988	1.777
2	2.296	1.915	3.445	2.911	7.028	4.322
3	3.656	2.988	5.697	5.627	11.076	7.803
	Story Drifts (‰)					
	0.5 x El Centro NS		1.0 x El Centro NS		1.5 x El Centro NS	
Story	PM	NL-RHA	PM	NL-RHA	PM	NL-RHA
1	2.001	1.765	2.998	2.510	7.470	4.441
2	3.739	3.076	5.614	5.277	10.100	7.316
3	3.400	2.904	5.630	6.865	10.120	9.442
	Plastic Rotations (rad)					
	0.5 x El Centro NS		1.0 x El Centro NS		1.5 x El Centro NS	
Story	PM	NL-RHA	PM	NL-RHA	PM	NL-RHA
1	0.002512	0.002022	0.004073	0.003249	0.008551	0.005161
2	0.002985	0.002343	0.005108	0.005659	0.009597	0.008109
3	0.002124	0.001714	0.004400	0.006031	0.008892	0.008720

overestimated, except the drift of third story for 1.0 x El Centro NS ground motion (underestimation 18%).

Locations of plastic hinges determined by PM and NL-RHA are identical. In particular, for 1.0 and 1.5 x El Centro NS excitations a plastic mechanism was created, while for 0.5 x El Centro NS plastic hinges were formed only at beams' ends. In Table 3 plastic rotations at

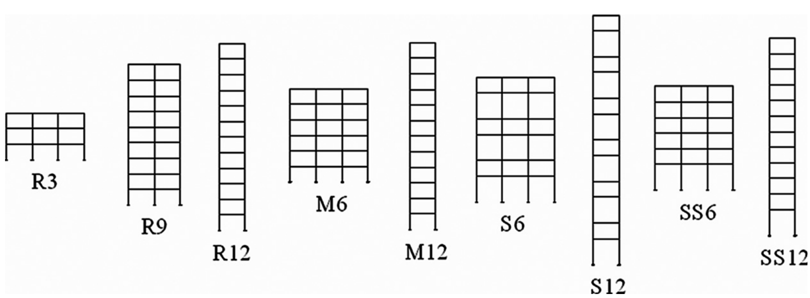


Figure 7. Geometrical scheme of the analyzed frames.

Table 4. Data of the analyzed frames

Data	Frames								
	R3	R9	R12	M6	M12	S6	S12	SS6	SS12
Story height (m)	3	3	3	3	3	3/5	3/5	3/5	3/5
Bay width (m)	5								
Concrete	C16/20 ($f_{ck} = 16$ MPa)								
Steel bars	S400 ($f_{yk} = 400$ MPa)								
Story mass (t)	30	30	15	20/40	9/16	25	10	30	13
Damping ratio (%)	5								
Gravity loads	Not Considered								
Beam cross-sections (cm)	25/40	25/50	25/50	25/40	25/50	25/40	25/50	25/40	25/50
Column cross-sections (cm)	40/40	60/60	60/60	50/50	60/60	50/50	60/60	50/50	60/60

critical sections of beams determined by PM and NL-RHA are also plotted. Notice that the maximum plastic rotations at the left and right end of each beam are equal, because the considered frame is symmetric (see also Step 7). As it is shown, with only one exception, the errors range between -25% and 25%.

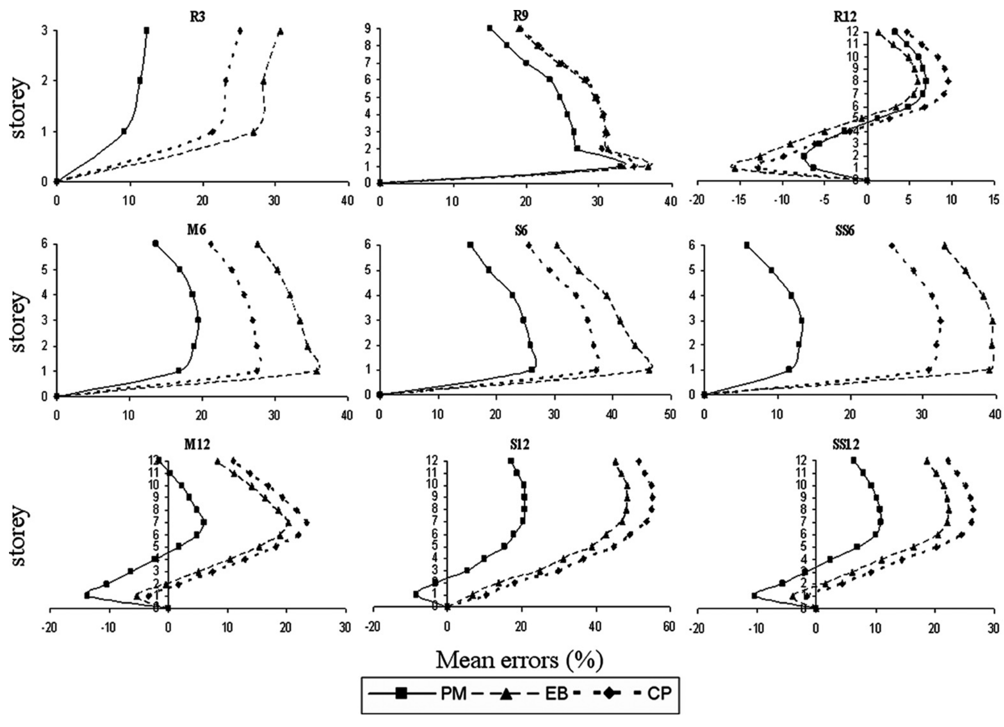


Figure 8. Mean errors (%) of story displacements.

STEP 7. APPLICATION OF STEPS 2-6 FOR FORCES ACTING IN THE OPPOSITE DIRECTION

Because of the symmetry of the structural model, it is not necessary to apply the lateral forces in both directions, so this step can be skipped.

EVALUATION OF THE PROPOSED METHODOLOGY

In order to evaluate the accuracy of the proposed methodology an extensive parametric study is carried out. In particular, the methodology is applied to a series of 3-, 6-, 9- and 12-story R/C planar frames designed according to old Greek codes (Figure 7, Table 4). Each frame is characterized by a string symbol comprising one or two letter(s) and a number which indicates the number of its stories. The meaning of the letter(s) is as follows:

- R – Regular frames
- M – Frames with irregular distribution of mass along the height (odd and even stories have different masses).
- S – Frames with irregular distribution of stiffness along the height (odd stories have greater height).
- SS – Frames with soft story (first story has greater height).

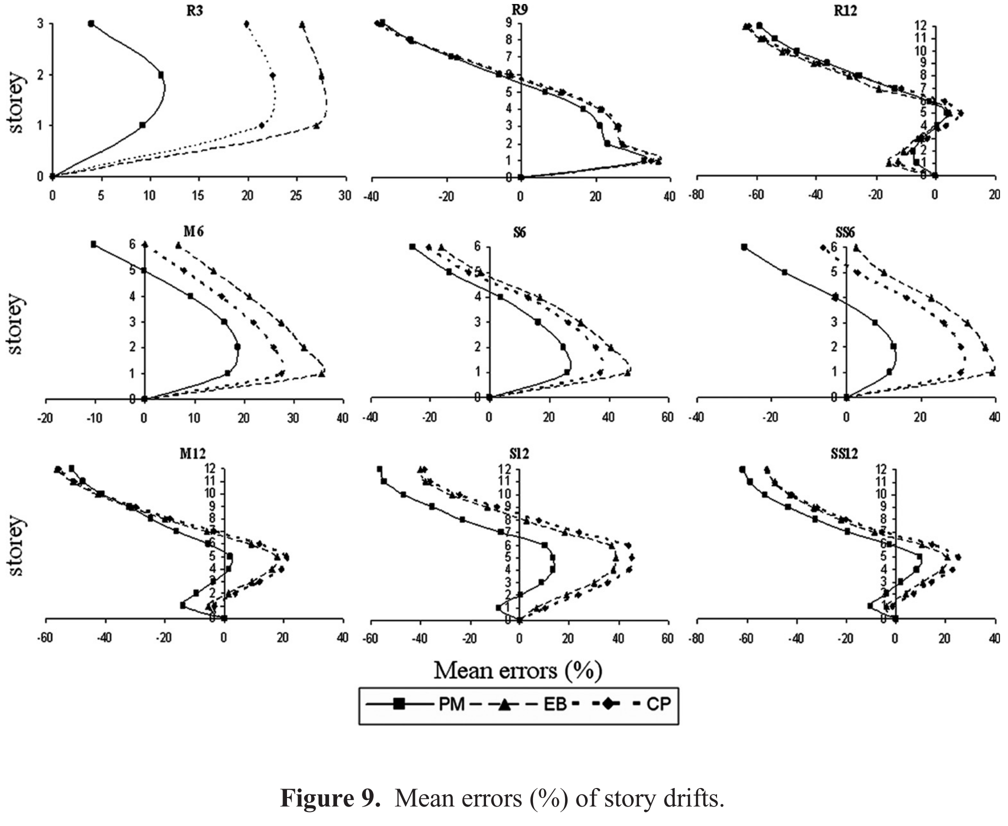


Figure 9. Mean errors (%) of story drifts.

For each frame three sets of pushover analyses are performed: i) one based on the proposed methodology (PM), ii) a second based on a procedure similar to the existing energy-based methods, i.e., according to it the energy equivalence between MDOF and E-SDOF systems is achieved by modifying the displacements (EB) and iii) a third based on the conventional displacement modification procedure (CP). The only difference between the three applied pushover procedures is the determination of the V_1-D_1 diagram (step 4), while the rest steps and assumptions are identical (see also the previous numerical example). V_1-D_1 diagram affects the characteristics of the E-SDOF system (particularly the proposed method leads to shorter T and greater R) and as a consequence the estimation of the target displacement. Each set of analyses comprises 12 different accelerograms corresponding to strong earthquake motions recorded in Greece. The maximum response of the E-SDOF system is calculated by means of nonlinear dynamic analysis for each excitation. Then, the target roof displacement is either estimated by multiplication of the resulting response by the quantity $v_1 \varphi_{N1}$ (PM, CP) or obtained by the roof displacement–energy-based displacement correspondence (EB) (Hernadez-Montes et al. 2004).

The story displacements and drifts of the frames under consideration are compared with those obtained by nonlinear response history analysis, which is considered as the reference solution. In Figures 8 and 9 the mean errors for the 12 excitations (in relevance to the NL-RHA results) of story displacements and drifts are shown. Notice that the positive sign (+) means that the response parameters obtained by NSPs are greater than those obtained by nonlinear time-history analysis. Conversely, the negative sign (-) means that the response parameters are underestimated by NSPs. From Figures 8 and 9 becomes clear that the proposed concept for the determination of the E-SDOF system leads to more accurate estimation of the target roof displacement (only in the case of frame R12, EB gives a little lower mean error). Mean errors range from -1% to 17% for PM, from 1% to 45% for EB and from 5% to 52% for CP. Concerning the rest response quantities, the mean errors resulting from the PM are sufficiently smaller in most cases (80% and 73% of cases in relevance to CP and EB respectively). All the three applied procedures fail to provide a reasonable estimation for drifts at the upper stories of taller frames. Such failures have been observed in many similar investigations due to the higher mode effects (e.g., Manoukas et al. 2006).

CONCLUSIONS

A new energy-based nonlinear static procedure (NSP) is presented in this paper. According to this procedure the determination of the characteristics of the E-SDOF system is based on a different concept with regard to the methods adopted by seismic codes. Specifically, the characteristics of the E-SDOF system are determined by equating the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system. This energy equivalence could be achieved by modifying either the displacement or the resisting force of the E-SDOF system. In contrast to other energy-based procedures, the proposed method follows the latter approach. The target displacement is then determined by using one of the well-known displacement modification procedures (e.g., ASCE 41-06). The preliminary evaluation of the proposed method shows that it leads to more accurate estimation of target roof displacement. Furthermore, in most cases the values of the remaining response parameters (story displacements and drifts) are more accurate too. None of the three applied pushover procedures can provide reasonable estimations of drifts at upper stories of tall buildings due to the higher modes effects. Conclusively, the whole investigation shows that, in general, the proposed methodology gives better results compared to those produced by the other applied procedures. However, the generalization of such conclusions is risky. In order to obtain secure generalized conclusions excessive investigations would be necessary comprising application of the proposed method to a large variety of structures and using an adequate number of earthquake ground motions.

For the present, the proposed method can be rigorously applied to low- and medium-rise planar frame structures with rather small contribution of higher mode effects. However, it can be easily expanded in a manner that allows its application to high-rise planar frames with significant contribution of higher modes, as well as to multistory asymmetric 3-D buildings. Relevant investigations are in progress and will be presented in a forthcoming paper.

Finally, it is worth noticing that the implementation of the proposed procedure in existing analysis software can be accomplished without particular difficulty.

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